# New experimental proposals for testing Dirac equation

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The advent of phenomenological quantum gravity has ushered us in the search for experimental tests of the deviations from general relativity predicted by quantum gravity or by string theories, and as a by–product of this quest the possible modifications that some field equations, for instance, the motion equation of spin–1/2–particles, have already been considered. In the present work a modified Dirac equation, whose extra term embraces a second–order time derivative, is taken as mainstay, and three different experimental proposals to detect it are put forward. The novelty in these ideas is that two of them do not fall within the extant approaches in this context, to wit, red–shift, atomic interferometry, or Hughes–Drever type–like experiments.

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One of the bedrocks beneath our present description of the fundamental laws of physics is embodied by Lorentz symmetry. The significance of this symmetry in the theoretical realm clearly justifies the long–lasting interest in testing it [1, 2, 3, 4]. One of the profits in this context, the one can be readily seen with a fleeting glimpse to the corresponding experimental constructions, is the fact that the involved precisions have undergone a remarkable improvement.

The struggle in the quest for a quantum theory of gravity, and the possibility of testing the different current approaches [5] have rendered some predictions about the modified field equations governing the motion of spin–1/2– particles, induced either by loop quantum gravity [6], or by string theory [7].

Amid the gamut of predicted effects we may find the presence of non–scalar mass terms, higher–order spatial derivatives, etc., [8]. Nevertheless, a thorough analysis in this context shall consider more general modifications to Dirac equation. For instance, the emergence of higher–order spatial derivatives must force us to mull over the appearance of higher–order time derivatives as part of a physically relevant possibility. It is in this last topic that the present work will delve. Forsooth, a second–order time derivative term will be considered as a primordial part of Dirac equation, and three new experimental proposals, whose intention is the detection of this additional contribution, will be put forward. Not only these ideas are independent from each other, but also two of them do not fall within the extant approaches in this context, to wit, red–shift, atomic interferometry, or Hughes–Drever type–like experiments [8].

The first idea addresses the dependence, upon the group velocity, of the spreading of a wave packet. It will be shown that, in principle, it is possible to detect higher–order time derivatives monitoring the so–called spreading time of a wave packet.

The second proposal will take advantage of the fact that the corresponding probability density displays a dependence, not only, upon the added term, but also upon the sign of the electric charge of the considered particle, a trait absent in the usual theory.

Finally, in the last idea we will use the fact that Larmor precession is, as will be shown later, a function of the extra term, and in consequence the angular velocity of the expectation values of the components of the spin allow us, in principle, to test our modified Dirac equation.

In addition the feasibility of implementing in an experimental effort each one of the proposed models is also, briefly, addressed.

As has been previously mentioned, our mainstay is the introduction of a second–order time derivative in Dirac equation. To wit, from square one we assume the following motion equation

$$i\hbar\frac{\partial}{\partial t}\psi = -i\hbar c\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}\psi + \beta mc^2\psi + \epsilon\frac{\hbar^2}{mc^2}\frac{\partial^2}{\partial t^2}\psi\,. \qquad (1)$$

A factor $1/mc^2$ in the term containing the second–order time derivative has been introduced, and the reason for this lies in the convenience of having a dimensionless parameter $\epsilon$. It is also readily seen that for $\epsilon = 0$, the introduced equation reduces to the usual Dirac situation. Additionally, a fleeting glimpse to (1) shows us that Lorentz–covariance is violated.

This kind of modified Dirac equation has already been considered [8], and also some experimental proposals for the detection of the new contribution have been put forward. At this point it is noteworthy to comment that all the aforementioned experiments fall within the realm of interferometry, red–shift, or atomic spectroscopy [8].

In the usual Dirac equation the non–relativistic limit is deduced by splitting up the energy into two parts, namely, (i) the rest energy, and (ii) additional contri-

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butions to the energy. This is attained introducing

$$\psi = \tilde{\psi}\exp\Big(-\frac{i}{\hbar}mc^2t\Big)\,. \tag{2}$$

The non–relativistic limit is obtained assuming that the rest energy is much larger than any other kind of energy involved. Proceeding as usual [9], which means that here

$$\tilde{\psi} = \begin{pmatrix} \phi \\ \chi \end{pmatrix}\,, \tag{3}$$

we arrive at the following expression

$$i[1-2\epsilon(1+\epsilon)]\hbar\frac{\partial\phi}{\partial t} = -\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\phi - \epsilon\frac{\hbar}{c}\lambda\frac{\partial^2\phi}{\partial t^2} + mc^2\epsilon^2(2+\epsilon)\phi. \tag{4}$$

Here $\lambda$ denotes the Compton wavelength of the particle. The presence of the last term in (4) requires further explanation. Indeed, it is readily seen that we do not know the order of magnitude of $\epsilon$. In other words, even if $\epsilon$ is very small, the term $mc^2\epsilon^2(2+\epsilon)$ could have an order of magnitude similar to the remaining energies present in (4).

The introduction of spin is relevant, not only because it is a fundamental physical trait, but also because one of the proposals requires the interaction of a magnetic field with spin. Accordingly, now we write down the generalized Dirac equation, considering its interaction with an electromagnetic field, and afterwards, its corresponding Pauli equation will be derived.

The introduction of the coupling with an electromagnetic field is achieved resorting to the minimal coupling procedure [8]. Therefore the resulting equation reads

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\psi &= -i\hbar c\boldsymbol{\alpha}\cdot\Big(\boldsymbol{\nabla} - \frac{iq}{\hbar c}\boldsymbol{A}\Big)\psi + \beta mc^2\psi \\ &+ \epsilon\frac{\lambda\hbar}{c}\Big(\frac{\partial}{\partial t} - iq\Phi\Big)^2\psi + q\Phi\psi\,. \end{aligned} \tag{5}$$

In (5) we have introduced the vector potential, $\boldsymbol{A}$, and the scalar one, $\Phi$. The non–relativistic limit of this last expression renders the generalized Pauli equation

$$\begin{aligned} &i[1-2\epsilon(1+\epsilon)]\hbar\frac{\partial\phi}{\partial t} = \frac{\Big(-i\hbar\boldsymbol{\nabla} - (q/c)\boldsymbol{A}\Big)^2}{2m}\phi + q\Phi\phi \\ &+\epsilon\frac{\lambda\hbar}{c}\Big(\frac{\partial}{\partial t} - iq\Phi\Big)^2\phi + mc^2\epsilon^2(2+\epsilon)\phi + \frac{q}{mc}\boldsymbol{S}\cdot\boldsymbol{B}\phi \end{aligned} \tag{6}$$

Two new terms have been introduced in (6), to wit, the magnetic field, $\boldsymbol{B}$, and the spin operator, $\boldsymbol{S}$, respectively.

Let us now consider a solution to (4) in the form

$$\phi \sim \exp\Big[i\Big(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t\Big)\Big]. \tag{7}$$

This Ansatz allows us to cast (4) in the following form

$$[1-2\epsilon(1+\epsilon)]\hbar\omega = \frac{\hbar^2k^2}{2m} + \epsilon\frac{\hbar}{c}\lambda\omega^2 + mc^2\epsilon^2(2+\epsilon). \tag{8}$$

It is readily seen that this last expression defines $\omega$ as a function of $k$. Indeed,

$$\begin{aligned} &\omega(k) = \frac{1}{2\epsilon\lambda}\Big\{[1-2\epsilon(1+\epsilon)]c \pm \\ &c\sqrt{[1-2\epsilon(1+\epsilon)]^2 - 4\frac{\epsilon\lambda}{c\hbar}[mc^2\epsilon^2(2+\epsilon) + \frac{\hbar^2k^2}{2m}]}\Big\}. \end{aligned} \tag{9}$$

Quantum Mechanics [10] teaches us that group and phase velocity are defined by, $\nu_g = \frac{d\omega}{dk}$ and $\nu_p = \frac{\omega}{k}$, respectively. Taking into account (9) we obtain

$$\nu_g = \frac{\hbar k}{m}\Big\{[1-2\epsilon(1+\epsilon)]^2 - 4\frac{\epsilon\lambda}{c\hbar}[mc^2\epsilon^2(2+\epsilon) + \frac{\hbar^2k^2}{2m}]\Big\}^{-1/2}. \tag{10}$$

An interesting point concerning the consequences of (9) is cognate with the fact that it defines a cutoff in the permitted wave number. Forsooth, the square–root, in (9), entails, in order to have real–valued frequency, the following condition

$$k \leq \sqrt{\frac{2m}{\hbar^2}}\Big\{\frac{c\hbar}{4\epsilon\lambda}[1-2\epsilon(1+\epsilon)]^2 - mc^2\epsilon^2(2+\epsilon)\Big\}^{1/2}. \tag{11}$$

Assuming $|\epsilon| << 1$, the cutoff, in terms of the momentum becomes, approximately

$$p \leq \frac{mc}{\sqrt{6\epsilon}}. \tag{12}$$

A condition always fulfilled within the non–relativistic realm. Consider now a one–dimensional wave packet constructed as a superposition of plane waves, in such a way that this packet is sharply peaked around $k = k_0$, with a width given by $\Delta k$

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} A(k-k_0)\exp\Big\{ikx - i\omega t\Big\}dk. \tag{13}$$

The condition upon the manner in which this wave packet has been constructed implies that $A(k-k_0) \approx 0$ if $|k-k_0| > \Delta k$. Expanding $kx - wt$ around $k = k_0$ allows us to cast (13) in the following form

$$\begin{aligned} \psi(x,t) &= \exp\Big\{ik_0x - i\omega(k_0)t\Big\}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} A(q) \\ &\times \exp\Big\{iq\Big(x - [\nu_g - q\frac{d^2\omega}{dk^2}_{|k_0}]t\Big)\Big\}dq. \end{aligned} \tag{14}$$

Here we have defined $q = k - k_0$. Since it has been assumed from the very beginning that $A(k-k_0) \approx 0$ if $|k-k_0| > \Delta k$, then (14) will be dominated by values of $q$ in the range $[-\Delta k, \Delta k]$. Hence, we are allowed to put forward the following relation

$$q\frac{d^2\omega}{dk^2}_{|k_0} = \pm\Delta\nu_g. \tag{15}$$

Knowing that the Fourier transform is dominated by those parts satisfying the condition $x - \nu_g t \approx 0$ (as long

as $(\Delta k)^2\frac{d^2\omega}{dk^2}_{|k_0}t << 1$), then it is reasonable to define the spreading time of the wave packet as

$$t_s = \Big[(\Delta k)^2\frac{d^2\omega}{dk^2}_{|k_0}\Big]^{-1}. \tag{16}$$

To first order in $\epsilon$ this spreading time reads

$$t_s = \frac{m}{\hbar(\Delta k)^2}\Big\{1-2\epsilon\Big[1-\frac{\lambda\hbar k_0^2}{2mc}\Big]\Big\}. \tag{17}$$

Let us now hark back to (6), with the initial assumption of vanishing magnetic field, namely, $\boldsymbol{B}=0$. Proceeding in the usual manner [10] it is possible to deduce a probability conservation law associated to (6). Indeed, under these circumstances

$$\frac{\partial\rho}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{J} = 0, \tag{18}$$

with

$$\begin{aligned}\rho &= \Big\{1-\epsilon\frac{q\Phi\lambda}{c[1-2\epsilon(1+\epsilon)]}\Big\}\phi\phi^* - i\epsilon\frac{\lambda}{c[1-2\epsilon(1+\epsilon)]}\\ &\times\ \Big\{\phi\frac{\partial\phi^*}{\partial t} - \phi^*\frac{\partial\phi}{\partial t}\Big\},\end{aligned} \tag{19}$$

and

$$\begin{aligned}\boldsymbol{J} &= i\frac{\hbar}{2m[1-2\epsilon(1+\epsilon)]}\Big[\phi\boldsymbol{\nabla}\phi^* - \phi^*\boldsymbol{\nabla}\phi\Big]\\ &- q\frac{\lambda}{\hbar[1-2\epsilon(1+\epsilon)]}\boldsymbol{A}\phi\phi^*.\end{aligned} \tag{20}$$

If $\epsilon = 0$ is implemented, then everything reduces to the usual conservation law [11]. The probability density not only hinges upon first–order time derivatives, it also displays a dependence on the charge of the involved particle. Both characteristics are absent in the usual model [11].

Let us now analyze the case in which spin has to be considered, and see if there is, in this context, enough leeway to pose an experimental proposal that could detect the extra term. As shown previously, the non–relativistic limit is embodied by (6). Henceforth it will be assumed that our involved particle is at rest and that the magnetic field has non–vanishing component only along the $z$–axis, i.e., $\boldsymbol{B} = B_0\boldsymbol{k}$, where $B_0$ is a constant with dimensions of magnetic field, and $\boldsymbol{k}$ denotes the unit vector along the $z$–axis. Under these restrictions the dynamics of the spin part of the system reads (here we have written the spin state ket as $|\chi>=\alpha|+> + \beta|->$, where $S_z|\pm>=\pm\frac{\hbar}{2}|\pm>$)

$$i[1-2\epsilon(1+\epsilon)]\hbar\frac{d\alpha}{dt} = -\epsilon\lambda\frac{\hbar}{c}\frac{d^2\alpha}{dt^2} + \frac{q\hbar}{2mc}B_0\alpha, \tag{21}$$

$$i[1-2\epsilon(1+\epsilon)]\hbar\frac{d\beta}{dt} = -\epsilon\lambda\frac{\hbar}{c}\frac{d^2\beta}{dt^2} - \frac{q\hbar}{2mc}B_0\beta. \tag{22}$$

It is readily seen that the solutions to these equations are (to second order in $\epsilon$) given by

$$\begin{aligned}|\chi> &= \cos\Big(\frac{\theta}{2}\Big)\exp\Big\{-i\frac{qB_0}{2mc}[1+2\epsilon(1+\epsilon)]t\Big\}|+>\\ &+ \sin\Big(\frac{\theta}{2}\Big)\exp\Big\{i\frac{qB_0}{2mc}[1+2\epsilon(1+\epsilon)]t\Big\}|->.\end{aligned} \tag{23}$$

In the last expression $\theta$ depends upon the initial conditions of the spin state ket. The condition $\epsilon = 0$ renders the usual situation [11]. If now the expectation value for $S_x$ is evaluated we find that

$$<S_x>_\chi = \frac{\hbar}{2}\sin\Big(\theta\Big)\cos\Big\{\frac{qB_0}{mc}[1+2\epsilon(1+\epsilon)]t\Big\}. \tag{24}$$

From (24) the frequency of this modified Larmor precession is easily read off

$$\omega = \frac{|q|B_0}{mc}[1+2\epsilon(1+\epsilon)]. \tag{25}$$

Let us now address the feasibility of the aforementioned experimental proposals. Firstly, the possibility of resorting to the spreading time of a wave packet in order to detect an extra term, like the one encompassed by (1), is cognate with the fact that the experimental resolution, $\Delta t$, has to be smaller than the difference between the spreading times in our proposal, (17), and the spreading time in the usual model, henceforth denoted by $\tilde{t}_s$, where $\tilde{t}_s = \frac{m}{\hbar(\Delta k)^2}$. In other words, it will be possible to detect, within the realm of the first proposal, an extra term like the one here considered if

$$\Delta t < \frac{2m}{\hbar(\Delta k)^2}\Big\{1-\frac{\lambda\hbar k_0^2}{2mc}\Big\}|\epsilon|. \tag{26}$$

This last expression may be used to set a bound, in the case of a null experiment, to the magnitude of $\epsilon$. Forsooth, if an experiment renders no evidence of this kind of extra term, then it means that

$$|\epsilon| < \frac{\hbar(\Delta k)^2}{2m}\Big\{1-\frac{\lambda\hbar k_0^2}{2mc}\Big\}^{-1}\Delta t. \tag{27}$$

Usually [8] the tests (which employ as probes quantum systems) of the postulates behind general relativity are divided into three different types: (i) Hughes–Drever type–like ideas, (ii) red–shift experiments, and (iii) interferometry. The latter is sensitive to the center of mass motion of quantum systems, whereas the former probes the energy of bound states. Clearly, the spreading time of a wave packet has no classical analogue, and in consequence the first proposal is a new test of Lorentz covariance, the one is not encompassed by neither of the three aforementioned ideas.

**Beware, new material inserted here.**

Let us now put forward a particular experimental setup designed to detect, within the context of spreading time, the magnitude of $\epsilon$.

Consider a particle at rest, whose wave function embodies a linear superposition of plane waves, in such a way that its initial form is gaussian (the maximum of the norm of the wave function will be at the origin of the coordinate system). Two screens will be located at two different points, such that they initially lie outside the root–mean square–deviation in the corresponding space variable. In other words, if the positions of the aforementioned screens are denoted by $0 < S_1 < S_2$ and $\Delta x(t = 0)$ is the root–mean–square–deviation at time $t = 0$, then $\Delta x(t = 0) < S_1$. As time goes by the packet spreads, and in consequence a time will come, say $t_1$, in which $\Delta x(t = t_1) = S_1$. Screen 1, at this moment, emits a photon. The same situation will be associated to the second screen, to wit, at time $t_2$, the root–mean–square–deviation fulfills $\Delta x(t = t_2) = S_2$. The time interval between these two photons will be related to the spreading velocity of the packet, and since we know the distance between the two screens, $S_2 - S_1$, then the knowledge of these two factors would allow us to set a bound to the $\epsilon$ parameter, the one appears in the spreading time, and in consequence in the spreading velocity. The possibility of measuring time intervals down to 50 fs is already within the technological developments. The experimental method is founded upon a fourth–order interference technique between two photons, and it permits the presence of an accuracy of 1 fs [12].

Therefore, we may reduce the measuring of the spreading velocity of the wave packet to the measuring of the time interval between two photons, which is a case that nowadays can be done with a very good precision [12].

**Beware, new material finishes here.**

The possibility of employing the probability density to detect the extra term is related to the fact that, as (19) clearly displays, the probability density hinges upon the charge of the corresponding particle, whereas in the usual theory it does not. Therefore, if we perform the change $q \to -q$, then the aforementioned expression leads us to conclude that there must be a change in the probability density. This change in the probability density associated to the modification of the charge of the involved particle is not present in the usual situation, and defines a trait that could, in principle lead to the detection of the new term.

For the sake of clarity let us assume that in our experiment we prepare the system such that $\phi\frac{\partial\phi}{\partial t}^* - \phi^*\frac{\partial\phi}{\partial t} = 0$, at $t = 0$. It would be possible to detect the extra term if, here $\Delta\rho$ denotes the experimental resolution in the measuring process of the probability density

$$\Delta\rho < |q\epsilon|\frac{\Phi\lambda}{c[1 - 2\epsilon(1 + \epsilon)]}\phi\phi^*. \tag{28}$$

Additionally, $\rho$, in the present model, has a time–dependence, embodied in the last term depicted in (19), the one does not emerge in the usual theory. The concept of probability density has not been used to detect any kind of violation to Lorentz covariance, and a fleeting glimpse to the current proposals [8] readily shows us that the second proposal does not fall within the usual experimental ideas.

**Beware, new material inserted here.**

Let us now introduce the possibility of detecting $\epsilon$ with the interaction embodied in (6). In order to do this we hark back to this aforementioned expression and take a very particular case, namely, we choose $\boldsymbol{A} = 0$. Henceforth, the dynamics does not embrace the spin of our particle, the one to first order in $\epsilon$ has the following face

$$\begin{aligned} i[1 - 2\epsilon(1 + \epsilon)]\hbar\frac{\partial\phi}{\partial t} &= \frac{\Big(-i\hbar\boldsymbol{\nabla}\Big)^2}{2m}\phi + q\Phi\phi \\ &+\epsilon\frac{\lambda\hbar}{c}\Big(\frac{\partial}{\partial t} - iq\Phi\Big)^2\phi. \end{aligned} \tag{29}$$

A fleeting glimpse to (4) (keeping only terms of first order in $\epsilon$) clearly shows us that we may interpret the presence of the term $[1 - 2\epsilon(1 + \epsilon)]$ as a redefinition of the inertial mass parameter as follows

$$\tilde{m} = m[1 - 2\epsilon(1 + \epsilon)]. \tag{30}$$

It is a very know fact that scattering of particles has been a useful tool in physics. Indeed, a lot of the most important discoveries in physics have been achieved with the help of this method [13]. The idea in this part of the work is to take advantage of the experience within this context, and try to put forward a physical quantity that could be measured, and which should render information about $\epsilon$. In this spirit, we may confront (29) against experimental evidence noting that in a scattering experiment, in the low–energy limit, the Born approximation entails the presence of the inertial mass parameter [14] for the scattering amplitude

$$f(\theta, \Phi) = -\frac{m}{2\pi\hbar^2}\int V(\vec{r})d^3\vec{r}. \tag{31}$$

The comment regarding the redefinition of the inertial mass parameter leads us to conclude that in the generalized Schrödinger equation the corresponding scattering amplitude (to first order in $\epsilon$) becomes (for spherical symmetry)

$$f(\theta) = -\frac{2m[1 - 2\epsilon]}{\hbar^2\kappa}\int_0^\infty rV(r)\sin(\kappa r)dr. \tag{32}$$

In this last equation we have introduced an additional parameter, to wit, $\kappa = 2k\sin(\theta/2)$. The connection with the experiment is deduced immediately recalling that the differential cross section $\frac{d\sigma}{d\Omega}$ is given by

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2. \tag{33}$$

To first order in $\epsilon$ we have that (here $\frac{d\sigma}{d\Omega}_U$ denotes the differential cross section in the usual model)

$$\frac{d\sigma}{d\Omega} = [1 - 4\epsilon]\frac{d\sigma}{d\Omega}_U. \qquad (34)$$

The proposed experiment could be carried out using electrons, in the long–wavelength limit, which should impinge upon a spherically symmetric scattering potential. This kind of experiments, as has been mentioned above, comprise already a good deal of experience, and in consequence this proposal lies within the present technological possibilities. The current precision associated to the detection of the number of particles scattered off would then define a bound to the magnitude of $\epsilon$.

**Beware, new material finishes here.**

Finally, a modified Larmor precession entails an additional manner to detect $\epsilon$. Looking at (25) it is easily seen that (in the usual case the Larmor frequence reads $\tilde{\omega} = \frac{qB_0}{mc}$ [11]) within this idea we need a time resolution, $\Delta t$, fulfilling

$$\Delta t < 2\epsilon\frac{mc}{|q|B_0}\Big(1 - \epsilon\Big). \qquad (35)$$

**Beware, new material inserted here.**

The feasibility of this last idea is cognate with the current technological precision related to the measurement of the so called Bohr frequency. Indeed, Larmor expression appears for the frequency of an atom, immersed in a uniform magnetic field, related to the energy eigenvalues of the Hamiltonian which describes the spin evolution [15]. Hence, the idea at this point is to exploit this fact, and in consequence in this last part of the present work the proposed experiment consists in the measurement of the energy difference of this kind of atoms, for instance, a silver atom, which is a system already studied within this realm. Indeed, denoting by $E_+$ and $E_-$ the two corresponding levels it is readily seen that the present idea leads us to look for deviations in the silver atom for the aforementioned energy difference, which in our case is tantamount to ($\Delta E_U = \hbar\tilde{\omega}/(2\pi)$ denotes the energy difference in the usual theory [16])

$$\Delta E = \Delta E_U[1 + 2\epsilon]. \qquad (36)$$

Though the extant literature already comprises results that evaluate the shift in the energy levels, for instance, of a hydrogen atom, our proposal involves the effects of the new term upon spin, a fact that seems to require further analysis. The question regarding the feasibility of the present proposal poses no difficulty, since this kind of experiments in spectroscopy have been already carried out [17]).

**Beware, new material finishes here.**

Summing up, quantum gravity and string theories entail possible modifications to some field equations, and in this realm our initial premise has been a modified Dirac equation, which embraces a second–order time derivative. The main idea in the present work delves with the detection of the aforementioned new term putting forward three new experimental proposals. At this point it is noteworthy to mention that two of them do not fall within the usual cases, either atomic interferometry, red shift, or Hughes–Drever type–like experiments. Finally, it is also feasible to detect this kind of modifications to Dirac equation looking at the changes that emerge in the context of Berry's phase. The results in this issue will be published elsewhere.

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